First French-Italian meeting on masonry
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Mechanics of Masonry Structures:
Micro, Multiscale and Macro Models

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Outline of the talk

1. MOTIVATIONS

2. MODELS FOR MASONRY
   • micromechanical models
   • phenomenological macro models
   • multiscale models

3. CONCLUSIONS
MOTIVATIONS
• Europe (and not only) is rich of historical buildings, monumental structures and heritage sites.

• The monumental heritage represents the history and the culture of the Country; it must be saved and preserved and (when necessary consolidated).

• Most of the heritage constructions are in masonry.
map of the seismic risk in Italy
IRPINIA

1980

via palladino
palazzo nisivoccia
basilica interno
Umbria-Marche 1997

Foligno: Piazza della Repubblica

October 14, 1997 05:23 pm
bell tower of the Town Hall.
What is the masonry material?

Masonry: heterogeneous material

mixture of (at least) two materials

natural stones, bricks

+ lime or cement based mortar

dry masonry
MODELS FOR MASONRY
Masonry modeling

Micro-mechanics

Phenomenological model

Homogenization

Discrete model

Continuum model

Macro-elements
Micromechanical models
Micromechanical models

- different **constitutive laws** for units and the mortar;
- structural analysis performed **considering each constituent** of the masonry material;
- mortar joints modeled as **interfaces** and bricks characterized by a linear or nonlinear response;
- structural analyses characterized by **great computational effort**; FEM: unit blocks and the mortar beds discretized, high number of nodal unknowns.

Lofti and Shing, 1994; Giambanco and Di Gati, 1997; Gambarotta and Lagomarsino, 1997; Lourenço and Rots, 1997; Giambanco et al., 2001; Oliveira and Lourenço, 2004; Alfano and Sacco, 2006; Fouchal et al. 2009; ....

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Analysis of masonry arch

Geometry:
• $R_b = 516$ mm
• $h = 12$ mm  $b = 5.5$ mm

Mechanical properties:
• $E = 8300$ N/mm$^2$
• $G = 3400$ N/mm$^2$
four hinges mechanism
Numerical simulation
• Brick/mortar:
  ➢ linear elastic / elastoplastic

• Brick-mortar interface: cohesive model
  ➢ damage,
  ➢ unilateral contact,
  ➢ friction
Cohesive-zone model

Microscale (REV)

Representative Elementary Volume
Main idea for coupling interface damage and friction

- On the undamaged part: elastic interface law
- On the damaged part: unilateral-friction law

\[ \sigma = (1 - D) \sigma^u + D \sigma^d \]

\[ \sigma^u = K s \]
\[ \sigma^d = K \left[ s - (c + p) \right] \]

\[ D \in [0, 1] : \text{damage parameter} \]

- Kinematic compatibility: \( s^u = s^d = s \)

- Additive decomposition of \( s^d \): \( s^d = s^{de} + s^{di} \)

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unilateral contact vector

\[ c = h(s_N) \begin{cases} s_N \\ 0 \end{cases} \quad h(s_N) = 1 \text{ if } s_N \geq 0 \\
= 0 \text{ if } s_N < 0 \]

inelastic slip: classical Coulomb yield function

\[ \phi(\tau^d) = \mu \langle \sigma_N^d \rangle - |\tau_T^d| = \mu \tau_N^d + |\tau_T^d| \]

\[ \dot{p}_T = \dot{\lambda} \frac{d\phi}{d\tau_T^d} = \dot{\lambda} \frac{\tau_T^d}{|\tau_T^d|} \]

\[ \dot{\lambda} \geq 0, \quad \phi(\sigma^d) \leq 0, \quad \dot{\lambda} \phi(\sigma^d) = 0 \]

damage

\[ \text{Area} = G_{cl} \]

ratios

\[ \eta_N = \frac{s_N^0}{s_N^f} = \frac{s_N^0 \tau_N^0}{2G_{cn}} \\
\eta_T = \frac{s_T^0}{s_T^f} = \frac{s_T^0 \tau_T^0}{2G_{ct}} \]

\[ \alpha = \sqrt{\langle s_N \rangle^2 + \langle s_T \rangle^2} \]

\[ \eta = 1 - \frac{1}{\alpha^2} \left( \langle s_N \rangle^2 \eta_N + s_T^2 \eta_T \right) \]

damage evolution

\[ D = \max_{\text{history}} \left\{ \min \{1, \tilde{D}\} \right\} \]

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Problem: assigned a history of average strain \[ \mathbf{E} = 1/h \{ s_T, s_N \}^T \]
evaluate the average stress state \[ \Sigma = 1/V \int_V \{ \sigma_{NT}, \sigma_N \}^T dV \]
accounting for the crack growth, the unilateral contact and the friction

Linear elastic **constitutive laws** for the mortar and brick

\[ \sigma^m = C^m \varepsilon^m, \quad \sigma^b = C^b \varepsilon^b \]

**Unilateral contact**

\[ d_N \geq 0, \quad \sigma \leq 0, \quad d_N \sigma = 0 \]

**Friction**

\[ \phi(\tau, \sigma) = \mu \langle \sigma \rangle - |\tau| = \mu \sigma + |\tau| \]

\[ \dot{\mathbf{p}} = \dot{\lambda} \begin{bmatrix} d\phi \\ d\tau \\ 0 \end{bmatrix} = \dot{\lambda} \begin{bmatrix} \tau \\ 0 \end{bmatrix} \]

\[ \dot{\lambda} \geq 0, \quad \phi(\tau, \sigma) \leq 0, \quad \dot{\lambda} \phi(\tau, \sigma) = 0 \]

**Crack growth: damage evolution**

Based on the relative displacement \( d_{NT}, d_N \) Fracture Mech

\( \tau, \sigma \) relative displs & stresses at the crack mouths

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Three subproblems

p1) RVE subjected to $s$, i.e. to $E$; relative displacement at the crack $d^e$.

p2) relative displacement prescribed at the crack mouths $d^c = -d^e$.

p3) RVE subjected to a frictional sliding at the crack mouths $p = \begin{pmatrix} p_T \\ 0 \end{pmatrix}$.
By simple superposition of the three solutions, it is possible to recover any possible mechanical situation.

<table>
<thead>
<tr>
<th>Open crack</th>
<th>Closed crack with no-sliding s1+s2</th>
<th>Closed crack with sliding s1+s2+s3</th>
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</thead>
<tbody>
<tr>
<td>s1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solution</td>
<td>s1</td>
<td>s2</td>
</tr>
<tr>
<td>Average strain, $E$</td>
<td>$E^e$</td>
<td>$0$</td>
</tr>
<tr>
<td>Average stress, $\Sigma$</td>
<td>$\Sigma^e$</td>
<td>$\Sigma^c$</td>
</tr>
<tr>
<td>Stresses at the crack, $\tau \sigma$</td>
<td>$0$</td>
<td>$\tau^c \sigma^c$</td>
</tr>
<tr>
<td>Relative displacement at crack, $d$</td>
<td>$d^e$</td>
<td>$d^e = -d^e$</td>
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</tbody>
</table>
Mechanical response

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Limit load, experimental results and numerical results


Loading:

initial compression obtained by prescribing the vertical displacement of the top (vertical reactions 30KN);

vertical displacements kept constant during the analysis;

horizontal displacement of the top-right corner incremented left-ward.

Each half brick discretized with $2 \times 2$ 4-noded, plane stress elements with enhanced strains.

Interface elements placed on the brick/mortar and on the brick/brick interfaces, to simulate the possible failure of a brick.

Numerically computed horizontal reaction $F$ plotted vs prescribed horizontal displacement; comparison with the experimental data provided by Lourenco.
Crack path
Phenomenological models
Macromechanical models

• phenomenological constitutive laws for the masonry, derived performing tests on masonry, **without distinguishing the blocks and the mortar** behavior;

• **unable to describe** in detail some **micro-mechanisms** occurring in the damage evolution of masonry;

• very **effective** from a **computational** point of view for the structural analyses.

• **NO-TENSION MODEL**
  
  Heyman, 1966; Zienkiewicz et al., 1968; Di Pasquale, 1978; Romano and Romano, 1979; Baratta, 1982; Como and Grimaldi, 1982; Romano and Sacco, 1983; Giaquinta and Giusti, 1988; Del Piero, 1989; Sacco, 1990; Angelillo, 1993; Lucchesi et al. 1994; Luciano and Sacco, 1994; Alfano et al., 2000; Cuomo and Ventura, 2000; Marfia and Sacco, 2005; ….
Damage-plastic (nonlocal) model

- stress-strain relation
  \[ \sigma = (1 - D)^2 C \varepsilon^e \]
  \[ = (1 - D)^2 C (\varepsilon - \varepsilon^P) \]

- isotropic damage,
- uncoupled damage and plastic evolutions,
- Drucker-Prager plasticity with isotropic hardening.

softening \[\rightarrow\] strain and damage localization \[\times\] strong mesh dependency

nonlocal constitutive law

• damage limit function:

\[ F_{nl} = \left( \frac{Y_t}{Y_{0t}} + \frac{Y_c}{Y_{0c}} - 1 \right) - \left\{ (\alpha_t \alpha_t + \alpha_c \alpha_c) \left( \frac{Y_t}{Y_{0t}} + \frac{Y_c}{Y_{0c}} \right) + \left( \alpha_t \frac{K_t}{Y_{0t}} + \alpha_c \frac{K_c}{Y_{0c}} \right) \right\} D + h \left( \frac{\alpha_t}{Y_{0t}} + \frac{\alpha_c}{Y_{0c}} \right) \nabla^2 D \]

- \( Y_t \) eq. strain tension: elastic strain
- \( Y_c \) eq. strain compression: total strain

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• damage evolution

\[ \dot{F} \dot{\bar{D}} = 0 \]
\[ \dot{\bar{D}} \geq 0 \quad F \leq 0 \quad F \dot{\bar{D}} = 0 \]

Consistency
Kuhn-Tucker

\[ \frac{\partial F}{\partial \bar{D}} \dot{\bar{D}} + \frac{\partial F}{\partial \bar{Y}} \dot{\bar{Y}} + \frac{\partial F}{\partial \nabla^2 \bar{D}} \nabla^2 \dot{\bar{D}} = 0 \]

nonlocal term

with \[ \dot{\bar{D}} > 0 \]

partial differential equation
• plasticity limit function (Drucker-Prager):
  \[ F_P(\tilde{\sigma}, q) = 3J_2 + (\sigma_c - \sigma_t) I_1 - \sigma_c \sigma_t + q \]

• \( \tilde{\sigma} = \frac{\sigma}{(1 - D)^2} \) effective stress

• \( q = -\chi \alpha \) isotropic hardening force

• \( s_t, s_c \) compressive and tensile strengths

• \( I_1, J_2 \) 1\textsuperscript{st} stress, 2\textsuperscript{nd} deviatoric invariants
- Masonry walls

Mesh independence (wall1) and Structural response graphs are shown.
damage distribution

minimum principal stress distribution

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No-tension model

- Masonry material characterized by very low tensile strength with respect to the compression strength.

- **Assumptions:**
  - the tensile strength is zero,
  - indefinitely elastic in compression.

- **Consequences:**
  - convex strain energy function,
  - reversible constitutive law,
  - no-energy dissipation (unrealistic?),
  - uniqueness of solution stress
  - no-uniqueness of solution displacement
  - apparent simplicity,
  - not trivial numerical treatment.
No-tension model with limited compressive strength

- Kinematics
  \[ \varepsilon = \varepsilon + \kappa + p \]

- Stress-stress relation (isotropic)
  \[ \sigma = E \varepsilon \]

- Convex cone \( K \) of the admissible stresses:
  \[ K = \{ \sigma : \sigma_1 \leq 0, \sigma_2 \leq 0 \} \]
Fracture & plasticity for the Drucker-Prager type yield plastic locus
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Unreinforced structure (right)
Unreinforced structure (left)
Reinforced structure (left)
Behavior of the structure

Multiplier of the horizontal loading vs. Horizontal displacement of P (mm)

- reinforced
- unreinforced

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The no-tension elasto-plastic model allows to catch the main features of the mechanical response of the masonry.
Multiscale models
Multiscale models

- different constitutive laws for the units and the mortar joints; homogenization procedure;
- very appealing, rational way to get the stress-strain relationship of the masonry, accounting for the failure micro-mechanisms of the masonry;
- effective models, with reduced computational effort;
- nonlinear homogenization procedure required to recover a macro-model could induce some theoretical or computational difficulties.

Kralj et al., 1991; Pietruszczak and Niu, 1992; Gambarotta and Lagomarsino, 1997; Luciano and Sacco, 1997; Cecchi and Sab, 2002; Cecchi et al. 2005; Uva and Salerno, 2006; Milani et al., 2006; Massart et al, 2007; Sacco, 2009; ....
Santa Emerenziana Church, built in 1940-42 designed by architect Tullio Rossi
Motivations: micro-macro analysis

Main idea: to develop a procedure which avoids the nonlinear FE$^2$ (FEA at each iteration in each Gauss point of each element)

Simple damage model:

- the cracks occur only in the mortar material which behaves in a perfect elastic-brittle manner,
- the bricks are indefinitely elastic,
- the mortar thickness is small, so that the cracks can develop only vertically or horizontally,
- when a fracture starts to develop, a full failure of a mortar junction is supposed.
Possible damaged states of the masonry

<table>
<thead>
<tr>
<th>Paths</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>S1 S2</td>
<td>S3 S8</td>
<td>S1 S2</td>
<td>S4 S8</td>
<td>S1 S5</td>
</tr>
<tr>
<td>S1 S5</td>
<td>S3 S8</td>
<td>S1 S5</td>
<td>S6 S8</td>
<td>S1 S7</td>
</tr>
<tr>
<td>S1 S7</td>
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<td>S6 S8</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

S1    | S2    | S3    | S4    | S5    |
S3    | S4    |       |       |       |
S5    | S6    | S7    |       |       |
S7    |       |       | S8    |       |
Homogenization

- UC problem solved for each possible state
- averages of the local stresses evaluated in the UC
- averages of the local stresses evaluated in each mortar joint
- failure of the mortar joint evaluated using the Coulomb friction criterion

\[
\begin{align*}
    c_n - \sigma_n &> 0 & c_n & \quad n - \text{cohesion} \\
    |\tau| &< c_t - \mu \sigma_n & c_t & \quad t - \text{cohesion} \\
    \mu & \quad \text{friction}
\end{align*}
\]

Solved via finite element method
Structural computation for different values of the cohesion for $\mu=1$
**New approach**

- first order homogenization technique, Cauchy continuum at macro- and micro-scale
- disadvantages of the first order models:
  - absolute size of the microstructure not incorporated,
  - intrinsic assumption of uniformity of the macroscopic fields (not appropriate in critical regions characterized by high deformation gradients),
  - mesh-dependency with softening.

- **enhanced continua** → **Cosserat model for masonry**

- **Cosserat continuum at the macro-scale,**
  Cauchy continuum at the micro-scale.

Kouznetsova et al. (2004); Forest and Sab (1998); van der Sluis et al. (1999); Masiani, Rizzi and Trovalusci (1995); Masiani and Trovalusci (1996, 2003); Casolo (2006); Brasile et al. (2007); De Bellis et al. (2008); Bacigalupo, Gambarotta (2010), Addessi et al. (2010).

Macro-level BVP

Displacement vector

\[ U = \begin{bmatrix} U_1 & U_2 & \Phi \end{bmatrix}^T \]

Strain vector

\[ E = \begin{bmatrix} E_1 & E_2 & \Gamma_{12} & \Theta & K_1 & K_2 \end{bmatrix}^T \]

Stress vector

\[ \Sigma = \begin{bmatrix} \Sigma_1 & \Sigma_2 & \Sigma_{12} & Z & M_1 & M_2 \end{bmatrix}^T \]

Governing equations

\[ \begin{aligned} 
E &= DU 
\quad \text{in } \Omega \\
D^T \Sigma + B &= 0 
\quad \text{in } \Omega \\
U &= \bar{U} 
\quad \text{on } \partial \Omega_U \\
N \Sigma &= T 
\quad \text{on } \partial \Omega_T 
\end{aligned} \]
Micro-level BVP

Displacement vector
\[ \mathbf{u} = \{u_1, u_2\}^T \]

Strain vector
\[ \mathbf{\varepsilon} = \{\varepsilon_1, \varepsilon_2, \gamma_{12}\}^T \]

Stress vector
\[ \mathbf{\sigma} = \{\sigma_1, \sigma_2, \tau_{12}\}^T \]

Governing equations
\[
\begin{align*}
\mathbf{\varepsilon} &= \mathbf{d} \mathbf{u} \\
\mathbf{d}^T \mathbf{\sigma} &= 0 \\
\mathbf{\sigma}^i &= \mathbf{F}^i (\mathbf{\varepsilon}) \\
& \text{in } \omega \\
& i = \text{mortar / brick}
\end{align*}
\]
Micro-Macro link

Displacement vector

\[ u = \tilde{u}(x) + \hat{u}(x) \]

Kinematical map

\[ \tilde{u} = A(x) \, E \]

Periodicity conditions

\[ \tilde{u} \text{ periodic on } \partial \omega \]
\[ n \sigma \text{ anti-periodic on } \partial \omega \]
Constitutive laws

brick model: linear elastic

mortar joint model: plasticity-damage-friction

\[ \sigma^M = C^M (\varepsilon - \pi) \]

\[ \pi = \begin{bmatrix} \pi_T \\ \pi_N \\ \pi_{NT} \end{bmatrix} = \begin{bmatrix} 0 \\ p_N \\ 0 \end{bmatrix} + D \begin{bmatrix} h(\varepsilon_N - p_N)\varepsilon_T \\ h(\varepsilon_N - p_N)(\varepsilon_N - p_N) \\ \gamma_{NT}^p \end{bmatrix} \]

crushing  damage  friction sliding (exponential law)
TFA: Transformation Field Analysis

Composite material

- Nonlinear effect in a region
- Constant inelastic strain in the region (Dvorak, 1992)
- Piecewise constant inelastic strain in the region (Chaboche, 2000)
- Non-uniform TFA (Michel-Suquet, 2003)
- TFA for damage / unilateral contact / friction (Sacco, 2009)
- Non-uniform TFA (Sepe-Marfia-Sacco, 2013)
Homogenization procedure (TFA)

8 sub-domains, where inelastic effects occur

Assumptions:
1. the inelastic strain $\mathbf{p}$ in each subset is uniform
2. the nonlinear behavior of the mortar is governed by the average stress in the subset
**Unit cell subjected to average elastic and to inelastic strains**

**elastic strain** $E_e$

localization of the strain $e(x) = R_e(x)E_e$

elastic strain in $M^j$ $e^{M^j} = R_e^{M^j}E_e$

average strain $E_e$

average stress (Hill-Mandel principle)

$$E_e^T \Sigma_e = \frac{1}{\Omega} \int_{\Omega} e^T \sigma \, d\Omega = \frac{1}{\Omega} E_e^T \left[ \int_B R_e^T C^B R_e \, d\Omega + \sum_{j=1}^{8} \int_{M^j} R_e^T C^{M^j} R_e \, d\Omega \right] E_e$$

$$= E_e^T C E_e \quad \Rightarrow \quad \Sigma_e = C E_e$$

**overall elastic matrix**
Unit cell subjected to average elastic and to inelastic strains

**Inelastic strain** \( \pi^i \)

Localization of the strain

\[ p^i(x) = R_{\pi^i}(x) \pi^i \]

Elastic strain in \( M^j \)

\[ \eta^i_{M^j} = \left( R_{\pi^i}^{M^j} - \delta_{ij} I \right) \pi^i \]

Average strain

\[ P^i = 0 \]

Average stress

\[
\Sigma_{\pi^i} = \frac{1}{\Omega} \left[ \int_{B} \left( R_{\pi^i}^{B} \right)^T C^{B} R_{\pi^i}^{B} \, d\Omega + \sum_{j=1}^{8} \int_{M^j} \left( R_{\pi^i}^{M^j} \right)^T C^{M^j} \left( R_{\pi^i}^{M^j} - \delta_{ij} I \right) \, d\Omega \right] \pi^i = S^i \pi^i
\]
Strain field localization

\[ E_e \quad \pi^i \]
\[ e(x) = R_e(x)E_e \quad p^i(x) = R_{\pi^i}(x)\pi^i \]

\text{average stress} \quad \Sigma_e = CE_e \quad \Sigma_{\pi^i} = S^i\pi^i \quad \text{total stress} \quad \Sigma = CE_e + \sum_{j=1}^{8} S^i\pi^i
\text{average strain} \quad E_e \quad 0 \quad \text{total strain} \quad E = E_e

Overall stress-strain relationship

\[ \Sigma = C(E - P) \quad P = -\sum_{j=1}^{8} C^{-1}S^i\pi^i \]

overall inelastic strain

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masonry wall loaded in compression and shear studied by Lourenço
• Homogenization:
  – Macro-Cosserat / Micro-Cauchy
  – Damage-friction cohesive model for the mortar
  – TFA technique
  – FEM for periodic masonry arrangements

• Cosserat components strongly affect the nonlinear behavior by influencing the damage initiation and evolution and the friction plastic flow

• Relevance of the use of the micro-polar Cosserat continuum for developing accurate models for masonry.
enhanced TFA
in elastic strain
\[ \pi^i = \sum_{k=1}^{h} \pi_k \phi_k^i(x) \]
\[ = \pi_0 + x_1 \pi_1 + x_2 \pi_2 + x_1 x_2 \pi_3 \]
CONCLUSIONS
Micro-mechanics

Discrete model
Masonry modeling

- Micro-mechanics
- Phenomenological model
  - Homogenization
    - Discrete model
    - Continuum model
  - Macro-elements
Thanks for your attention