The MIMO Antenna as a Communication Channel

Marco D. Migliore
(1)

(1) DAEIMI, University of Cassino, Cassino, Italy, http://www.unicas.it

Introduction

The understanding of the connections between information theory and electromagnetic theory is a new topic which is attracting a fast growing number of researchers [1]-[5]. This work represents a further contribution toward the understanding of the connections between these two theories. In particular, in this contribution the antenna is discussed from an usual point of view, in which the antenna itself is treated as a communication channel. This approach allows to understand in a simple way how antennas affect the throughput of a MIMO system. Furthermore, this approach seems to be particularly useful for identifying the intrinsic informational limitations of a MIMO antenna.

The communication channel approach to MIMO antennas

We consider the model of a “pure spatial” communication channel introduced in [2], properly modified to consider a stochastic environment. Since in this communication system information are coded in the spatial variation of the electromagnetic field, instead of temporal variation, this model is particularly suitable to study the behaviour of communication systems in the spatial domain.

Fig 1: MIMO system geometry

The input (Fig 2) represents the primary excitations of the TX antennas. This quantity is the input of the “source distribution” system S, the output of which is the current density J in the space. The current density J is the input of the operator A modelling the radiating process relating J to the field E on a given observation manifold. The measurement system M allows to obtain the measured quantity y. Paper [2] was focused on the identification of the informational limits regarding sources with a given spatial extension, and the operator M (that models the receiving antenna F and (spatially) AWG noise n) was supposed the identity operator plus noise. This work is instead focused on the informational limits regarding the antenna e.g the operator F. In the following we will suppose that F is a linear operator, e.g. the antenna does not include any non-linear devices, like...
real amplifiers and mixers. Furthermore, we suppose that the receiving antenna is spatially limited by the curve $\Omega$ (Fig 1).

The domain of $F$ is the set of all the possible electromagnetic fields radiated by the sources and incident on the antenna. Paper [2] shows that in the case of a fixed scenario the radiation operator $A$ is compact and the incident field belongs to a practically finite dimensional space provided that the observation manifold does not intersect any current source. In the present paper the scenario belongs to an ensemble of scenarios which represents the set of all the possible environments in which the antenna is supposed to operate. Consequently, the incident field is a stochastic process, and can be approximated in mean using the Karhounen-Loève (K-L) expansion [6]. The kernel of the K-L operator is the spatial covariance function of the incident field on $\Omega$, equal to the autocorrelation function under the hypothesis of zero mean. Under the hypothesis that the kernel is square integrable, the K-L integral operator is compact, and it is possible to parallel the analysis performed in paper [2]. In particular, the incident field can be approximated (in mean) at any degree of accuracy using a basis whose dimension turns out the minimum one for the required degree of approximation. The dimension of such a space will be called the Statistical Number of Degrees of Freedom (StNDF) of the incident field, paralleling the definition of the NDF of the field [7]. It can be easily demonstrated that the compactness of the radiation operator $A$ assures the compactness of the K-L operator in any practical case, including the case of uncorrelated source distribution current $J$. The analysis of the K-L expansion on the observation domain shows that the StNDF is a function of the spatial autocorrelation of the source distribution current $J$. Furthermore, the NDF is an upperbound for the StNDF, and coincides with the NDF in the case of spatially uncorrelated current density sources $J$. Under the hypothesis of jointly gaussian distribution, the coefficients of the K-L expansion are also statistically independent so that they can convey a number of StNDF statistically independent information. Consequently, the system $O$ in Fig 2 can be seen as an informational source, while the transmitted signal is a point in a signal space whose effective dimension is equal to StNDF. This space will be called the “incident signal space”. For the sake of simplicity, in the following we will suppose that $x$ is a standard circularly symmetric complex gaussian vector, e.g. $x = CN(0, I)$, wherein $I$ is the StNDF $\times$ StNDF identity matrix.
Let us discuss now the role of the receiving antenna. The antenna is a linear operator mapping the $StNDF$ dimensional incident signal space into the received signal space. Since the dimension of the received signal space is finite-dimensional due to the finite number of active elements of the antenna, the antenna can be represented by a finite-dimensional operator $F$ and $y = Fx + n$ (see Fig 3). By expanding $F$ by means of Singular Value Decomposition we have $y' = \Sigma x' + n'$ (see Fig 4), wherein $F = U \Sigma V^H$, $U$ is a matrix whose columns are the left singular vectors of $F$, $V$ is a matrix whose columns are the right singular vectors, $\Sigma$ is a diagonal matrix whose elements are the singular values $\sigma_k$ of $F$, $y' = U^H y$, $x' = V^H x$, and $n' = U^H n$. Since $U$ is a unitary matrix, if $x = CN(0, I)$, also $x' = CN(0, I)$. With reference to Fig 3, $z' = CN(0, \Sigma^2)$. By supposing that $n = n' = CN(0, N_0 I)$, the mutual information $I(y, x)$ is equal to

$$I(y', x') = \sum_{k=1}^{r} \log_2 \left( 1 + \frac{\sigma_k^2}{N_0} \right),$$

wherein $r$ is the rank of $F$. The expression of the mutual information shows that only the terms associated to the singular values having a square amplitude significantly higher than the noise variance $N_0$ give a significant contribution.

From the above discussion, it is clear that the rank $r$ is limited by $\min(StNDF, N)$, wherein $N$ is the number of active elements of the antenna. However, there is a further constraint in the rank $r$. In fact, let us consider a generic receiving antenna placed inside $\Omega$. Applying the reciprocity theorem, it is possible to evaluate the maximum $NDF$ of the field radiated by any radiator placed in the observation manifold limited by $\Omega$. In particular, we consider a receiving curve surrounding the radiation source and placed at least at a couple of wavelengths from the source. The $NDF$ of the field radiated on such a curve is a function of only the radiating source, and not of the particular observation curve. In the following this number will be called $NDF_{\Omega}$ (e.g. the maximum $NDF$ associated to any antenna placed on $\Omega$), and for not superdirective antenna it is of the order of twice the electrical length of $\Omega$ (e.g. twice the length of $\Omega$ measured in wavelength) [3]. The practical consequence of $NDF_{\Omega}$ is the following: independently of the particular operator $F$ (e.g. of the specific antenna), no more than $NDF_{\Omega}$ degrees of freedom can be discriminated by the antenna placed on $\Omega$. This means that no more than $NDF_{\Omega}$ statistically independent coefficients can be resolved by any antenna placed on $\Omega$. Consequently, $r$ is not greater than $\min(StNDF, N, NDF_{\Omega})$.

Conclusions

The antenna can be seen as a communication channel. According to this point of view, the throughput of the system depends (also) on the mutual information between the “electromagnetic incident field source” and the antenna output signal.
By the point of view of antenna synthesis, it is of interest to find the “channel” operator $F$ which maximizes the mutual information among the set of operators verifying the physical and design constraints on the antenna. However, since the solution depends also on $x$ covariance matrix, a practical approach for the antenna synthesis is an adaptive synthesis, which dynamically chooses the $F$ operator within a set of available operators taking into account the environment in which the antenna is working, as happens for examples in AdaM antennas [8], [9].

A further interesting observation is that the effective dimension of the domain and the range of the antenna operator $F$ is not greater respectively than $NDF_\Omega$ and $N$. This means that a MIMO antenna can be fully characterized by not more than $NDF_\Omega \times N$ parameters. Such number of parameters is an intrinsic property of the antenna, and is independent of the environment in which the antenna operates. This result is the key point of a method to characterize MIMO antennas currently under development at the University of Cassino.

References: